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Baryon acoustic signature in the clustering of density maxima

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Baryon acoustic signature in the clustering of density maxima

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We reexamine the two-point correlation of density maxima in Gaussian initial conditions. Spatial derivatives of the linear density correlation, which were ignored in the calculation of Bardeen, Bond, Kaiser & Szalay (1986), are included in our analysis. These functions exhibit large oscillations around the sound horizon scale for generic CDM power spectra. We derive the exact leading-order expression for the correlation of density peaks and demonstrate the contribution of those spatial derivatives. In particular, we show that these functions can modify significantly the baryon acoustic signature of density maxima relative to that of the linear density field. The effect depends upon the exact value of the peak height, the filter shape and size, and the small-scale behaviour of the transfer function. In the Λ CDM cosmology, for a threshold height $\nu = 2$ and a Gaussian window of characteristic scale $\approx 8 \times 10^{13} \text{ M}_\odot/h$ for instance, the contrast of the BAO is twice as large as in the linear matter correlation. Overall, the BAO is amplified for $\nu \gtrsim 1$ and damped for $\nu \lesssim 1$. Density maxima thus behave quite differently than linearly biased tracers of the density field, whose acoustic signature is a simple scaled version of the linear baryon acoustic oscillation. We also demonstrate the first order cancellation of the mean streaming of peak pairs. This suggests that some of the enhancement of the BAO in the primeval correlation of density maxima may survive in the correlation of high redshift density peaks. Galaxy biasing will be an important issue in ascertaining how much of this effect propagates into the late-time clustering of galaxies.

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I. INTRODUCTION

Sound waves propagating in the primordial photon-baryon fluid imprint an oscillatory pattern in the anisotropies of the Cosmic Microwave Background (CMB) and in the matter distribution, whose characteristic length scale r_s is the sound horizon at the recombination epoch [1]. $r_s \approx 105 h^{-1} \text{ Mpc}$ for the currently favoured cosmological models. While experiments have accurately measured this fundamental scale and its harmonic series in the temperature and polarisation power spectra of the CMB, this acoustic signature has recently been detected in the correlation function of galaxies [2, 3]. There is also weak evidence for the baryon oscillations in the correlation function of clusters [4]. In the two-point correlation, the series of maxima and minima present in the power spectrum translate into a broad peak at the sound horizon scale. Since the latter can be accurately calibrated with CMB measurements, the baryon acoustic oscillations (BAO) have emerged as a very promising standard ruler for determining the angular diameter distance and Hubble parameter [5]. Measuring the BAO at different redshifts thus offers a potentially robust probe of the dark energy equation of state.

In linear theory, the amplitude of the baryon acoustic peak increases while its shape and contrast remain unchanged. However, the clustering of galaxies does not fully represent the primeval correlation. Mode-coupling, pairwise velocities and galaxy bias are expected to alter the position and shape of the acoustic peak and, therefore, bias the measurement [6]. The evolution of the acoustic pattern in the two-point statistics of the matter, halo or galaxy distributions has been studied us-

ing both numerical simulations [7, 8] and analytic techniques based on the halo model or perturbation theory [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. Yet the results of these studies do not always agree and the impact of nonlinearities on the matter and galaxy power spectrum remains debatable. For instance, [11, 13] argue that any systematic shift (i.e. not related to random motions or biasing) must be less than the percent level owing to the particularly smooth power added by nonlinearities on those scale, and to the cancellation of the mean streaming of (linearly) biased tracers at first order. On the other hand, [12, 14, 16, 17] have shown that mode-coupling modifies the acoustic pattern in the correlation of dark matter and haloes, and generates a percent shift towards smaller scales. Despite their redshift dependence, these shifts appear to be predictable and could be removed from the data [20].

There is a broad consensus regarding the shape of the acoustic peak. In light of the nonlinear gravitational evolution of matter fluctuations, it is sensible to expect a baryon acoustic peak less pronounced in the late-time clustering of galaxies than in the linear theory correlation. This can be shown to hold for any local biasing mechanism [13, 23, 24]. Models based on local bias do indeed predict a damping of the baryon acoustic features in the two-point statistics of the galaxy distribution [8, 9, 14]. Galaxies, of course, form a discrete set of points but one commonly assumes them to be a Poisson sample of some continuous field. However, whether local bias is an accurate approximation to the clustering of galaxies remains unclear. Notice also that a reconstruction of the primordial density field could significantly restore the original contrast of the acoustic oscillation [22].

The main objective of this paper is to demonstrate that the baryon acoustic signature in the correlation of tracers of the density field can be noticeably modified if we drop the assumption of local bias. To this purpose, we will examine the clustering of density maxima in the initial cosmological density field. Density peaks form a well-behaved point-process whose statistical properties depend not only on the underlying density field, but also on its first and second derivatives. Therefore, although the number density of maxima is modulated by large-scale fluctuations in the background, their clustering properties cannot be derived from a local (deterministic or stochastic) bias model in which the peak density would depend upon local properties of the matter density alone. Furthermore, we shall assume that the initial fluctuations are described by Gaussian statistics. This assumption is remarkably well supported by measurements of the CMB and large-scale structures [25, 41].

In a seminal paper, Bardeen, Bond, Kaiser & Szalay (hereafter BBKS) [26] provided a compact expression for the average number density of peaks in a three-dimensional Gaussian random field etc. Furthermore, they obtained a large-scale approximation for the correlation function of peaks which, at large threshold height, tends toward the correlation of overdense regions [27, 28, 29] as it should be. However, BBKS determined the correlation function of density maxima only in the limit where derivatives of the two-point function of the density field can be ignored. As we will see below, these correlations can greatly influence the large-scale correlation of density maxima for generic Cold Dark Matter (CDM) power spectra. It is also worthwhile noticing that the statistics of Gaussian random field in a cosmological context has received some attention in the literature [30, 31, 32, 33, 34]. Some of these results have been applied to the mass function and correlation of rich clusters for example [35, 36, 37]. The present work mainly follows the analytic study of BBKS, and the lines discussed in [38, 39], where two-point statistics of the linear tidal shear are investigated. We refer the reader to [40] for a rigorous introduction to the statistics of maxima of Gaussian random fields.

The paper is organised as follows. Section §II introduces a number of useful variables and correlation functions. Section §III is devoted to the derivation of the leading order expression for the large-scale asymptotics of the peak correlation. In §IV, this result is applied to illustrate the impact of spatial derivatives of the density field on the amplitude and shape of the correlation of density maxima. Our attention focuses on the baryon oscillation, across which the amplitude of the linear matter correlation varies abruptly. It is shown that the BAO of density maxima can be amplified relative to that of the matter distribution. Section §V demonstrates that the relative pairwise velocity of peaks vanishes at first order, suggesting thereby that the impact of nonlinear evolution should be at the few percent level. A final section summarises our results.

II. PROPERTIES OF COSMOLOGICAL GAUSSIAN DENSITY FIELDS

We review some general properties of Gaussian random fields and provide explicit expression for the correlations of the density and its lowest derivatives. We show that the latter are not always negligible in CDM cosmologies.

A. Definitions

We will assume a Λ CDM cosmology with normalisation amplitude $\sigma_8 = 0.82$, and spectral index $n_s = 0.96$ [41]. The matter transfer function is computed using publicly available Boltzmann codes [42]. The position of the BAO in the linear matter correlation function is close to $\approx 105.0 h^{-1}\text{Mpc}$. Furthermore, $\delta_{\text{sc}} = 1.673$ and $M_\star \approx 3.5 \times 10^{12} M_\odot/h$ at redshift $z = 0$.

Let \mathbf{q} designate the Lagrangian coordinate. We are interested in the three-dimensional density field $\delta(\mathbf{q})$ and its first and second derivatives. It is more convenient to work with the normalised variables $\nu = \delta(\mathbf{q})/\sigma_0$, $\eta_i = \partial_i \delta(\mathbf{q})/\sigma_1$ and $\zeta_{ij} = \partial_i \partial_j \delta(\mathbf{q})/\sigma_2$, where the σ_j are the spectral moments of the matter power spectrum,

$$\sigma_j^2 \equiv \int_0^\infty d\ln k k^{2j} \Delta^2(k). \quad (1)$$

$\Delta^2(k) \equiv \Delta_\delta^2(k) |\hat{W}(k, R_f)|^2$ denotes the dimensionless power spectrum of the density field smoothed on scale R_f with a spherically symmetric window $\hat{W}(k, R_f)$. The best choice of smoothing is open to debate. Among the popular window functions, the tophat filter is compactly supported and has a straightforward interpretation within the spherical collapse model. Notwithstanding this, oscillations that arise in Fourier space do not lead to well defined spectral moments σ_j with $j \geq 2$ for CDM power spectra. This can be understood by examining the high- k tail of the CDM transfer function. Neglecting the baryon thermal pressure on scale less than the Jeans length, the small-scale matter transfer function behaves as $T(k) \propto \ln(1.8k)/k^2$ [26, 43], which clearly leads to divergences when the integer j is larger than one. By contrast, a Gaussian window function ensures the convergence of all the spectral moments for any realistic matter power spectra. Consequently, we shall mostly rely on the Gaussian filter throughout this paper, although the tophat filter will also be considered briefly in Section §IV. Note that a Gaussian filter of characteristic width R_f encloses a mass $M_f = (2\pi)^{3/2} \bar{\rho} R_f^3$ a few times larger than that encompassed by a tophat filter of identical smoothing radius.

Following BBKS, we also introduce the parameters $\gamma = \sigma_1^2/(\sigma_0\sigma_2)$ and $R_\star = \sqrt{3}\sigma_1/\sigma_2$ for subsequent use. The spectral width γ reflects the range over which $\Delta^2(k)$ is large, while R_\star characterises the radius of peaks. For the special case of a powerlaw power spectrum with Gaussian filtering on scale R_f , these parameters are given by $\gamma^2 =$

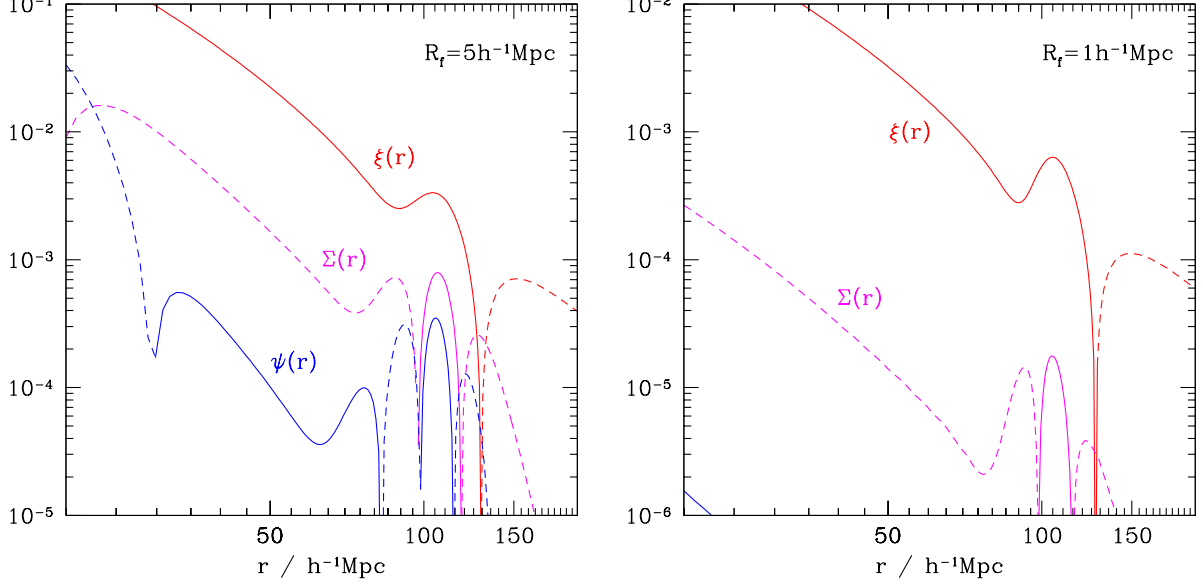


FIG. 1: A comparison between the cross-correlation of the density field, $\xi(r)$, and that of its first and second derivatives, $\Sigma(r)$ and $\psi(r)$ respectively (see text). Results are shown as a function of the Lagrangian separation r for the Λ CDM cosmology considered in the present work. In the left panel, the density field is smoothed with a Gaussian filter of characteristic scale $R_f = 5 h^{-1}\text{Mpc}$ (e.g. a mass scale $M_f = 1.5 \times 10^{14} M_\odot/h$) whereas, in the right panel, the smoothing length is $R_f = 1 h^{-1}\text{Mpc}$ ($M_f = 1.2 \times 10^{12} M_\odot/h$). In the latter case, the correlation function $\psi(r)$ (not shown) is less than 10^{-6} at distances larger than $\gtrsim 30 h^{-1}\text{Mpc}$. Dashed lines denote negative values. All the correlations are normalised to unity at zero lag.

$(n+3)/(n+5)$ and $R_*^2 = 6R_f^2/(n+5)$. For CDM power spectra, $\gamma \sim 0.5 - 0.7$ when the smoothing length varies over the range $0.1 \lesssim R_f \lesssim 10 h^{-1}\text{Mpc}$.

B. Correlation of the density and its derivatives

Calculating the two-point correlation of density peaks requires knowledge of the auto- and cross-correlations of the various fields. These objects can be decomposed into components with definite transformation properties under rotations. Statistical isotropy and symmetry implies that, in position space, the most general ansatz for the isotropic sector of the two-point correlations of these fields reads

$$\begin{aligned}
 \langle \nu(\mathbf{q}_1) \nu(\mathbf{q}_2) \rangle &= \xi(r) \\
 \langle \nu(\mathbf{q}_1) \eta_i(\mathbf{q}_2) \rangle &= \Xi(r) \hat{r}_i \\
 \langle \nu(\mathbf{q}_1) \zeta_{ij}(\mathbf{q}_2) \rangle &= -\gamma \Sigma_1(r) \hat{r}_i \hat{r}_j - \gamma \Sigma_2(r) \delta_{ij} \\
 \langle \eta_i(\mathbf{q}_1) \eta_j(\mathbf{q}_2) \rangle &= \Sigma_1(r) \hat{r}_i \hat{r}_j + \Sigma_2(r) \delta_{ij} \\
 \langle \eta_i(\mathbf{q}_1) \zeta_{lm}(\mathbf{q}_2) \rangle &= \Pi_1(r) \hat{r}_i \hat{r}_l \hat{r}_m + \Pi_2(r) (\hat{r}_i \delta_{lm} + \hat{r}_l \delta_{im} + \hat{r}_m \delta_{il}) \\
 \langle \zeta_{ij}(\mathbf{q}_1) \zeta_{lm}(\mathbf{q}_2) \rangle &= \Psi_1(r) \hat{r}_i \hat{r}_j \hat{r}_l \hat{r}_m \\
 &\quad + \Psi_3(r) (\hat{r}_i \hat{r}_l \delta_{jm} + \hat{r}_i \hat{r}_m \delta_{jl} + \hat{r}_j \hat{r}_l \delta_{im} + \hat{r}_j \hat{r}_m \delta_{il} \\
 &\quad + \hat{r}_i \hat{r}_j \delta_{lm} + \hat{r}_l \hat{r}_m \delta_{ij}) + \Psi_5(r) (\delta_{ij} \delta_{lm} + \delta_{il} \delta_{jm} + \delta_{im} \delta_{jl}),
 \end{aligned} \tag{2}$$

where $r = |\mathbf{q}_2 - \mathbf{q}_1|$ is the Lagrangian separation, $\hat{r}_i = r_i/r$ and the functions ξ , Ξ , Σ_i , Π_i and Ψ_i depend on

r only. We emphasise that these correlation functions transform as scalar under rotations. Note also that these expressions are valid for any arbitrary random field. For a cosmological Gaussian density field however, these functions can be summarised as follows :

$$\begin{aligned}
 \xi(r) &= \frac{1}{\sigma_0^2} \int_0^\infty d\ln k \Delta^2(k) j_0(kr) \\
 \Xi(r) &= -\frac{1}{\sigma_0 \sigma_1} \int_0^\infty d\ln k k \Delta^2(k) j_1(kr) \\
 \Sigma_1(r) &= -\frac{1}{\sigma_1^2} \int_0^\infty d\ln k k^2 \Delta^2(k) j_2(kr) \\
 \Sigma_2(r) &= \frac{1}{\sigma_1^2} \int_0^\infty d\ln k k^2 \Delta^2(k) \left[\frac{1}{3} j_0(kr) + \frac{1}{3} j_2(kr) \right] \\
 \Pi_1(r) &= -\frac{1}{\sigma_1 \sigma_2} \int_0^\infty d\ln k k^3 \Delta^2(k) j_3(kr) \\
 \Pi_2(r) &= \frac{1}{\sigma_1 \sigma_2} \int_0^\infty d\ln k k^3 \Delta^2(k) \left[\frac{1}{5} j_1(kr) + \frac{1}{5} j_3(kr) \right] \\
 \Psi_1(r) &= \frac{1}{\sigma_2^2} \int_0^\infty d\ln k k^4 \Delta^2(k) j_4(kr) \\
 \Psi_3(r) &= -\frac{1}{\sigma_2^2} \int_0^\infty d\ln k k^4 \Delta^2(k) \left[\frac{1}{7} j_2(kr) + \frac{1}{7} j_4(kr) \right] \\
 \Psi_5(r) &= \frac{1}{\sigma_2^2} \int_0^\infty d\ln k k^4 \Delta^2(k) \\
 &\quad \times \left[\frac{1}{15} j_0(kr) + \frac{2}{21} j_2(kr) + \frac{1}{35} j_4(kr) \right].
 \end{aligned} \tag{3}$$

$j_\ell(x)$ are spherical Bessel functions of the first kind. In the limit $r \rightarrow 0$, all the correlation functions vanish but ξ , Σ_2 and Ψ_5 , which tend towards 1, $1/3$ and $1/15$, respectively. Averaging over the direction $\hat{\mathbf{r}}$ of the separation vector thus yields

$$\begin{aligned} \frac{1}{4\pi} \int d\Omega_{\hat{\mathbf{r}}} \langle \eta_i(\mathbf{q}_1) \eta_j(\mathbf{q}_2) \rangle &= \frac{\Sigma(r)}{3} \delta_{ij} \\ \frac{1}{4\pi} \int d\Omega_{\hat{\mathbf{r}}} \langle \zeta_{ij}(\mathbf{q}_1) \zeta_{lm}(\mathbf{q}_2) \rangle &= \frac{\psi(r)}{15} (\delta_{ij} \delta_{lm} + \delta_{il} \delta_{jm} + \delta_{im} \delta_{jl}) \end{aligned} \quad (4)$$

for the covariances of the fields η_i and ζ_{ij} , where we have defined

$$\begin{aligned} \Sigma(r) &= \Sigma_1(r) + 3\Sigma_2(r) \\ \psi(r) &= \Psi_1(r) + 10\Psi_3(r) + 15\Psi_5(r). \end{aligned} \quad (5)$$

The angular average of the other correlation functions vanishes, except that of the density correlation of course.

$\Sigma(r)$ and $\psi(r)$ can be expressed in terms of the derivatives of the density correlation using relations like $\langle \eta_i \eta_j \rangle = -\partial_i \partial_j \xi(r)$ etc. For a density correlation that falls off as a powerlaw r^{-n-3} , $\Sigma(r)$ and $\psi(r)$ decay as r^{-n-5} and r^{-n-7} , respectively. This derivation assumes a powerlaw power spectrum with a fair amount of power at short wavenumbers. Hence, as recognised in BBKS, neglecting the derivatives of the density correlation should be a reasonable approximation when $n \lesssim -1$.

This simple argument may not hold for CDM cosmologies since the index n is a smooth function of the separation r . Namely, it is $n \sim -2$ when $r \sim 10 h^{-1}\text{Mpc}$, and increases to attain a value of the order of unity on scale $r \gtrsim 60 h^{-1}\text{Mpc}$. For illustration purpose, the functions ξ , Σ and ψ are plotted in Fig. 1 for the ΛCDM cosmology considered here. The filtering length is $R_f = 5$ (left panel) and $1 h^{-1}\text{Mpc}$ (right panel). Retaining only the density correlation appears to be a good approximation on scales larger than a few smoothing radii. However, the relative amplitude of the cross-correlations strongly depends upon the filtering scale. Namely, both $\Sigma(r)$ and $\psi(r)$ increase relative to $\xi(r)$ with increasing smoothing length. Yet another striking feature of Fig. 1 is the oscillatory behaviour of $\Sigma(r)$ and $\psi(r)$. The large oscillations are caused by rapid changes in the linear matter correlation across the baryon acoustic peak. Notice that both $\Sigma(r)$ and $\psi(r)$ are positive at distances $r \approx 100 - 110 h^{-1}\text{Mpc}$. On these scales, when $R_f = 1 h^{-1}\text{Mpc}$, $\Sigma(r)$ reaches to 3 per cent of the density correlation while $\psi(r)$ is negligible. At the large smoothing length however, they nearly reach 20 and 10 per cent of the density correlation, respectively.

These results suggest that, for generic CDM power spectra, the derivatives of the density correlation could have a significant impact on the correlation of density maxima, especially in the vicinity of the baryon acoustic feature. This motivates the calculation presented in the next Section.

III. CORRELATION OF DENSITY MAXIMA

Owing to the constraints on the derivatives of the density field, calculating the n -point correlation function of peaks requires performing integration over a joint probability distribution in $10n$ variables. Therefore, even the evaluation of the two-point correlation of density maxima $\xi_{\text{pk}}(r)$ proves difficult. Here, we derive the leading order expression that includes, in addition to the linear matter correlation $\xi(r)$, the contribution of the angular average functions $\Sigma(r)$ and $\psi(r)$. This result will be used in §IV to quantify the sensitivity of the acoustic signature of $\xi_{\text{pk}}(r)$ on the peak height and the amount of smoothing.

A. The Kac-Rice formula

As shown in BBKS for instance, the correlation of density extrema (maxima, minima and saddle points) can be entirely expressed in terms of $\delta(\mathbf{q})$ and its derivatives, $\eta_i(\mathbf{q})$ and $\zeta_{ij}(\mathbf{q})$. In the neighbourhood of an extremum, the first derivative η_i is approximately

$$\eta_i(\mathbf{q}) \approx \sqrt{3} R_\star^{-1} \sum_j \zeta_{ij}(\mathbf{q}_p) (\mathbf{q} - \mathbf{q}_p)_j. \quad (6)$$

Using a similar expansion for the field $\nu(\mathbf{q})$, we find that the number density of extrema can be written as

$$n_{\text{ext}}(\mathbf{q}) = \sum_p \delta^3(\mathbf{q} - \mathbf{q}_p) = \frac{3^{3/2}}{R_\star^3} |\det \zeta(\mathbf{q})| \delta^3[\eta(\mathbf{q})], \quad (7)$$

provided that the Hessian ζ_{ij} is invertible. The delta function $\delta^3[\eta]$ ensures that all the extrema are included. In this paper however, we are interested in counting the maxima solely. Consequently, we further have to require $\zeta_{ij}(\mathbf{q}_p)$ be negative definite at the extremum position \mathbf{q}_p . Note that, later, we will also restrict the set to those maxima with a certain threshold height. The average number density of maxima eventually reads

$$\langle n_{\text{pk}}(\mathbf{q}) \rangle = \frac{3^{3/2}}{R_\star^3} \langle |\det \zeta(\mathbf{q})| \delta^3[\eta(\mathbf{q})] \rangle. \quad (8)$$

This expression, known as the Kac-Rice formula [40, 44, 45, 46, 47], holds for arbitrary smooth random fields. In the special case of a Gaussian random field in three dimensions, it can be shown that the mean density of maxima scales as $\langle n_{\text{pk}} \rangle \propto R_\star^{-3} \propto R_f^{-3}$ [26].

The two-point correlation function of density peak is defined such that

$$1 + \xi_{\text{pk}}(\mathbf{r}) = \langle n_{\text{pk}}(\mathbf{q}_1) n_{\text{pk}}(\mathbf{q}_2) \rangle / \langle n_{\text{pk}} \rangle^2 \quad (9)$$

is the joint probability that a density maxima is in a volume dV_i about each \mathbf{q}_i . Let \mathbf{X} be the diagonal matrix of entries $\text{diag}(x_1, x_2, x_3)$ where $x_1 \geq x_2 \geq x_3$ is the non-increasing sequence of eigenvalues of the symmetric matrix $-\zeta$. The condition that the extrema are maxima

implies $x_3 \geq 0$. Therefore, the correlation function of peaks is given by

$$\begin{aligned} 1 + \xi_{\text{pk}}(\mathbf{r}) &= \frac{3^3}{\langle n_{\text{pk}} \rangle^2 R_\star^6} \langle |\det \zeta_1| |\det \zeta_2| \theta(x_3) \theta(y_3) \delta^3[\eta_1] \delta^3[\eta_2] \rangle \\ &= \frac{3^3}{\langle n_{\text{pk}} \rangle^2 R_\star^6} \int d\nu_1 d^6 \zeta_1 d\nu_2 d^6 \zeta_2 |\det \zeta_1| |\det \zeta_2| \theta(x_3) \theta(y_3) \\ &\quad \times P(\eta_1 = 0, \nu_1, \zeta_1, \eta_2 = 0, \nu_2, \zeta_2; \mathbf{r}) , \end{aligned} \quad (10)$$

where, for shorthand convenience, subscripts denote quantities evaluated at different Lagrangian positions, and $d^6 \zeta = \prod_{i \leq j} d\zeta_{ij}$ is the usual Lebesgue measure on the six-dimensional space of symmetric matrices. Here and henceforth $\theta(x)$ designates the Heaviside step-function, i.e. $\theta(x) = 1$ for $x > 0$ and zero otherwise.

B. Two-point probability distribution

The joint probability distribution of the density fields together with its first and second derivatives, $P(\eta_1, \nu_1, \zeta_1, \eta_2, \nu_2, \zeta_2; \mathbf{r})$, is given by a multivariate Gaussian whose covariance matrix \mathbf{C} has 20 dimensions. This 20×20 matrix may be partitioned into four 10×10 block matrices, $\mathbf{M} = \langle \mathbf{y}_1 \mathbf{y}_1^\top \rangle = \langle \mathbf{y}_2 \mathbf{y}_2^\top \rangle$ in the top left corner and bottom right corners, $\mathbf{B} = \langle \mathbf{y}_1 \mathbf{y}_2^\top \rangle$ and its transpose in the bottom left and top right corners, respectively. The components ζ_A , $A = 1, \dots, 6$ of the ten-dimensional vector $\mathbf{y}^\top = (\eta_i, \nu, \zeta_A)$ symbolise the entries $ij = 11, 22, 33, 12, 13, 23$ of ζ_{ij} . To emphasise that the entries ζ_A transform as a tensor under rotation, we shall also label them as the matrix ζ in what follows.

The matrices \mathbf{M} and \mathbf{B} can be further decomposed into block sub-matrices of size 4 and 6,

$$\mathbf{M} = \begin{pmatrix} \mathbf{M}_1 & \mathbf{M}_3^\top \\ \mathbf{M}_3 & \mathbf{M}_2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \mathbf{B}_1 & \mathbf{B}_3^\top \\ \mathbf{B}_3 & \mathbf{B}_2 \end{pmatrix}. \quad (11)$$

Unlike the \mathbf{M}_i which describe the covariances at a single position, the matrices \mathbf{B}_i generally are functions of the separation vector \mathbf{r} . Using the harmonic decomposition of the tensor products $\hat{\mathbf{r}} \otimes \dots \otimes \hat{\mathbf{r}}$, they can be written as

$$\begin{aligned} \mathbf{B}_1(\mathbf{r}) &= \mathbf{B}_1^{0,0} + \sum_{\ell=1}^4 \mathbf{B}_1^{\ell,m}(\mathbf{r}) Y_\ell^m(\hat{\mathbf{r}}) \\ \mathbf{B}_2(\mathbf{r}) &= \mathbf{B}_2^{0,0} + \sum_{\ell=1}^4 \mathbf{B}_2^{\ell,m}(\mathbf{r}) Y_\ell^m(\hat{\mathbf{r}}) \\ \mathbf{B}_3(\mathbf{r}) &= \mathbf{B}_3^{0,0} + \sum_{\ell=1}^4 \mathbf{B}_3^{\ell,m}(\mathbf{r}) Y_\ell^m(\hat{\mathbf{r}}). \end{aligned} \quad (12)$$

$Y_\ell^m(\hat{\mathbf{r}})$ are spherical harmonics and $\mathbf{B}_i^{\ell,m}(\mathbf{r})$ are matrices which satisfy $(\mathbf{B}_i^{\ell,m})^\dagger = (-1)^m \mathbf{B}_i^{\ell,m}$. Only multipoles up to $\ell = 4$ appear in the harmonic decomposition since the

correlations given in eq. (3) involve up to products of four unit vectors $\hat{\mathbf{r}}$. The monopole terms are

$$\begin{aligned} \mathbf{B}_1^{0,0} &= \begin{pmatrix} \Sigma(r)/3 \mathbf{I} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & \xi(r) \end{pmatrix}, \\ \mathbf{B}_2^{0,0} &= \begin{pmatrix} \psi(r)/15 \mathbf{A} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \psi(r)/15 \mathbf{I} \end{pmatrix}, \\ \mathbf{B}_3^{0,0} &= \begin{pmatrix} \mathbf{0}_{3 \times 3} & -\gamma \Sigma(r)/3 \mathbf{1}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} \end{pmatrix}, \end{aligned} \quad (13)$$

where

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}, \quad (14)$$

\mathbf{I} is the 3×3 identity matrix and $\mathbf{1}_{1 \times 3} = (1, 1, 1)$ etc. The matrices \mathbf{M}_i are readily obtained as $\mathbf{M}_i = \mathbf{B}_i^{0,0}(0)$. An explicit computation of the higher multipole matrices is unnecessary here as we confine the calculation to the monopole contribution.

It is important to note that the joint density $P(\mathbf{y}_1, \mathbf{y}_2, \mathbf{r})$ preserves its functional form under the action of the rotation group $\text{SO}(3)$. However, in a given frame of reference, $P(\mathbf{y}_1, \mathbf{y}_2, \mathbf{r})$ does change when $\hat{\mathbf{r}}$ moves on the unit sphere. Does this mean that $\xi_{\text{pk}}(\mathbf{r})$ truly depends on the direction of the separation vector \mathbf{r} ? No, as it should be clear from eq. (11) where the volume measure $|\det \zeta| d^6 \zeta$ is a rotational invariant. More precisely, the volume element $d^6 \zeta$ can be cast into the form

$$d^6 \zeta = 8\pi^2 |\Delta(x)| d^3 x d\mathbf{R}. \quad (15)$$

where the x_i s are, as before, the three ordered eigenvalues of $-\zeta$, $d^3 x = dx_1 dx_2 dx_3$ and $\Delta(x) = \prod_{i < j} (x_i - x_j)$ is the Vandermonde determinant. $d\mathbf{R}$ is the Haar measure (for the Euler angles for example) on the group $\text{SO}(3)$ normalised to $\int d\mathbf{R} = 1$. The peak correlation thus is proportional to

$$\int d\mathbf{R}_1 d\mathbf{R}_2 P(\eta_1 = 0, \nu_1, \zeta_1, \eta_2 = 0, \nu_2, \zeta_2, \mathbf{r}), \quad (16)$$

where the integral runs over the two $\text{SO}(3)$ manifolds that define the orientation of the principal frames of ζ_1 and ζ_2 relative to the frame of reference. Alternatively, we can choose the coordinate system such that the coordinate axes are aligned with the principal axes of ζ_1 . In this new coordinate frame, the above integral becomes

$$\frac{1}{4\pi} \int d\Omega_{\hat{\mathbf{r}}} d\mathbf{R} P(\eta_1 = 0, \nu_1, \zeta_1, \eta_2 = 0, \nu_2, \zeta_2, \mathbf{r}), \quad (17)$$

where \mathbf{R} is an orthogonal matrix that defines the orientation of the eigenvectors of ζ_2 relative to those of ζ_1 . This demonstrates that only the monopole component of $P(\mathbf{y}_1, \mathbf{y}_2, \mathbf{r})$ contributes to the peak correlation function. Therefore, $\xi_{\text{pk}}(\mathbf{r})$ is invariant under rotations of the coordinate system, namely, it is a function of the separation r only.

C. Large scale asymptotics

To obtain the correlation function of peak, we need first to calculate the two-point probability distribution function averaged over the unit sphere for the variables $\mathbf{y}^\top = (\eta_i, \nu, \zeta_A)$,

$$P(\mathbf{y}_1, \mathbf{y}_2, r) = \frac{1}{4\pi} \int d\Omega_{\hat{\mathbf{r}}} P(\mathbf{y}_1, \mathbf{y}_2, r). \quad (18)$$

In the large-distance limit ($r \gg 1$), the cross-correlation matrix is small when compared to the zero-point contribution M , e.g. $|B| \ll M$. Following [38, 39], the quadratic form which appears in the probability distribution $P(\mathbf{y}_1, \mathbf{y}_2; r)$,

$$P(\mathbf{y}_1, \mathbf{y}_2; r) = \frac{1}{(2\pi)^{10} |\det C|^{1/2}} e^{-Q(\mathbf{y}_1, \mathbf{y}_2, r)}, \quad (19)$$

where $\det C \approx |\det M|^2 = 4^2 (1 - \gamma^2)^2 / (15^{10} 3^8)$ is the determinant of the covariance matrix C , can be computed

easily using Schur's identities. Expanding the exponential in the small perturbation B yields, to first order,

$$e^{-Q(\mathbf{y}_1, \mathbf{y}_2, r)} \approx (1 + \mathbf{y}_1^\top M^{-1} B M^{-1} \mathbf{y}_2) e^{-\bar{Q}(\mathbf{y}_1, \mathbf{y}_2)}, \quad (20)$$

where the quadratic form $\bar{Q}(\mathbf{y}_1, \mathbf{y}_2)$ can be recast as

$$2\bar{Q} = \nu_1^2 + \frac{(\gamma\nu_1 + \text{tr}\zeta_1)^2}{1 - \gamma^2} + \frac{5}{2} [3\text{tr}(\zeta_1^2) - (\text{tr}\zeta_1)^2] + 1 \leftrightarrow 2, \quad (21)$$

in agreement with the results of BBKS. The calculation of $\mathbf{y}_1^\top M^{-1} B M^{-1} \mathbf{y}_2$ is tedious but straightforward. Fortunately, only the monopole terms $B_i^{0,0}$ survive after averaging over the directions $\hat{\mathbf{r}}$. After further simplification, the result can be reduced to the following compact expression :

$$\begin{aligned} \frac{1}{4\pi} \int d\Omega_{\hat{\mathbf{r}}} \mathbf{y}_1^\top M^{-1} B M^{-1} \mathbf{y}_2 &= \frac{5}{2} [3 \text{tr}(\zeta_1 \zeta_2) - \text{tr}\zeta_1 \text{tr}\zeta_2] \psi(r) + \{ \text{tr}\zeta_1 \text{tr}\zeta_2 [\psi(r) + \gamma^2 \xi(r)] \\ &+ \nu_1 \nu_2 [\xi(r) + \gamma^2 \psi(r)] - 2\gamma^2 (\text{tr}\zeta_1 \text{tr}\zeta_2 + \nu_1 \nu_2) \Sigma(r) + \gamma (\nu_1 \text{tr}\zeta_2 + \nu_2 \text{tr}\zeta_1) [\xi(r) + \psi(r) - (1 + \gamma^2) \Sigma(r)] \} (1 - \gamma^2)^{-2}. \end{aligned} \quad (22)$$

The invariance under rotation requires that $P(\mathbf{y}_1, \mathbf{y}_2, r)$ be a symmetric function of the eigenvalues, and thus a function of $\text{tr}(\zeta_1^k \zeta_2^l)$, $k, l = 0, 1, \dots$

Since the above expression depends only upon the relative orientation of the two principal axes frames of ζ_1 and ζ_2 (through the presence of $\text{tr}(\zeta_1 \zeta_2)$), we choose a coordinate system whose axes are aligned with the principal frame of ζ_1 . With this choice of coordinate, we define $\zeta_1 = -X$ and $\zeta_2 = -RYR^\top$, where R is an orthogonal matrix that defines the relative orientation of the eigenvectors of ζ_2 . X and Y are the diagonal matrices consisting of the three ordered eigenvalues x_i and y_i of the Hessian $-\partial_i \partial_j \nu$. The properties of the trace imply that $\text{tr}\zeta_1 = -\text{tr}X$, $\text{tr}(\zeta_1^2) = \text{tr}(X^2)$ (and similarly for ζ_2), while the term $\text{tr}(\zeta_1 \zeta_2) = \text{tr}(XRYR^\top)$ depends explicitly on the rotation matrix R .

The integral over the $SO(3)$ manifold that describes the orientation of the orthonormal triad of ζ_1 is immediate. The result is $2\pi^2$ (and not $8\pi^2$) as we don't care whether the axes are directed towards the positive or negative direction. The integral over the second $SO(3)$ manifold involves

$$\int_{SO(3)} dR \text{tr}(XRYR^\top) = \frac{1}{3} \text{tr}X \text{tr}Y, \quad (23)$$

and yields cancellation of the first term in the right-hand side of eq. (23). To integrate over the eigenvalues

of ζ_1 and ζ_2 , we transform to the new set of variables $\{u_i, v_i, w_i, i = 1, 2\}$, where

$$\begin{aligned} u_1 &= x_1 + x_2 + x_3 \\ v_1 &= (x_1 - x_3)/2 \\ w_1 &= (x_1 - 2x_2 + x_3)/2. \end{aligned} \quad (24)$$

The variables (u_2, v_2, w_2) are similarly defined in terms of the y_i . We will henceforth refer to u as the peak curvature.

Our choice of ordering impose the constraints $v_i \geq 0$ and $-v_i \leq w_i \leq v_i$. The condition that the density extrema be maxima, i.e. all three eigenvalues of the Hessian ζ_{ij} are negative, translates into $(u_i + w_i) \geq 3v_i$. Another condition, $u_i \geq 0$, should also be applied if one is interested in selecting maxima with positive threshold height.

For shorthand convenience, and to facilitate the comparison with the calculation of BBKS, we introduce the auxiliary function

$$\begin{aligned} F(u_1, v_1, w_1) &\equiv \frac{3^3}{2} |\det X| \Delta(x) \\ &= (u_1 - 2w_1) [(u_1 + w_1)^2 - 9v_1^2] v_1 (v_1^2 - w_1^2), \end{aligned} \quad (25)$$

$F(u_1, v_1, w_1)$ measures the degree of asphericity expected for a peak and can be used to determine the probability distribution of ellipticity v_1/u_1 and prolateness w_1/u_1 [26]. It scales as $\propto u_i^3$ in the limit $u_i \gg 1$.

D. The peak correlation $\xi_{\text{pk}}(\nu, r)$

We will henceforth focus on the cross-correlation $\xi_{\text{pk}}(\nu_1, \nu_2, r)$ between peaks of threshold height ν_1 and ν_2 as it is better suited to a comparison with, e.g., dark mat-

ter haloes extracted from N-body simulations or galaxies/clusters from a survey. $n_{\text{pk}} = n_{\text{pk}}(\nu)$ will hereafter denote the differential density of peaks in the range ν to $\nu + d\nu$. The expectation value of the product of the local peak densities that appears in eq. (9) is then

$$\xi_{\text{pk}}(\nu_1, \nu_2, r) = \frac{1}{\langle n_{\text{pk}} \rangle^2} \frac{5^5 3^4}{(2\pi)^6} R_\star^{-6} (1 - \gamma^2)^{-1} \int \prod_{i=1,2} \{ du_i dv_i dw_i F(u_i, v_i, w_i) \} \Phi_0(\nu_1, \nu_2, u_1, u_2, r) e^{-\bar{Q}}, \quad (26)$$

where

$$\begin{aligned} \Phi_0(\nu_1, \nu_2, u_1, u_2, r) = & \{ u_1 u_2 [\psi(r) + \gamma^2 \xi(r)] \\ & + \nu_1 \nu_2 [\xi(r) + \gamma^2 \psi(r)] - 2\gamma^2 (u_1 u_2 + \nu_1 \nu_2) \Sigma(r) - \gamma (u_1 \nu_2 + u_2 \nu_1) [\xi(r) + \psi(r) - (1 + \gamma^2) \Sigma(r)] \} (1 - \gamma^2)^{-2}, \end{aligned} \quad (27)$$

is equation (23) averaged over the relative orientation of the frames spanned by the eigenvectors of ζ_1 and ζ_2 . Φ_0 depends on the separation r through the correlation functions $\xi(r)$, $\Sigma(r)$ and $\psi(r)$ only. Furthermore, the quadratic form \bar{Q} simply is

$$2\bar{Q} = \nu_1^2 + \frac{(u_1 - \gamma \nu_1)^2}{1 - \gamma^2} + \frac{15}{2} \nu_1^2 + \frac{5}{2} w_1^2 + 1 \leftrightarrow 2 \quad (28)$$

in the variables (24).

The integration over the variables v_i and w_i is lengthy but straightforward. We refer the reader to BBKS for the details since the calculation now proceeds along similar lines. Let us mention that the allowed domain of integration is the interior of a triangle bounded by the

points $(0, 0)$, $(u_i/4, -u_i/4)$ and $(u_i/2, u_i/2)$. As shown in BBKS, the differential density of peak of height ν can be cast into the form

$$n_{\text{pk}}(\nu) = \frac{1}{(2\pi)^2 R_\star^3} e^{-\nu^2/2} G_0(\gamma, \gamma\nu), \quad (29)$$

where G_0 is the zeroth moment of the peak curvature u . Higher moments are written in explicit compact form as

$$G_n(\gamma, \omega) = \int_0^\infty dx x^n f(x) \frac{e^{-(x-\omega)^2/2(1-\gamma^2)}}{\sqrt{2\pi(1-\gamma^2)}}. \quad (30)$$

Using this result, the correlation of peaks can be rearranged as follows :

$$\xi_{\text{pk}}(\nu_1, \nu_2, r) = G_0(\gamma, \gamma\nu_1)^{-1} G_0(\gamma, \gamma\nu_2)^{-1} \int \prod_{i=1,2} \left\{ du_i f(u_i) \frac{e^{-(u_i - \gamma \nu_i)^2/2(1-\gamma^2)}}{\sqrt{2\pi(1-\gamma^2)}} \right\} \Phi_0(\nu_1, \nu_2, u_1, u_2, r). \quad (31)$$

For sake of completeness,

$$\begin{aligned} f(x) = & \frac{1}{2} (x^3 - 3x) \left\{ \text{Erf} \left[\sqrt{\frac{5}{2}} x \right] + \text{Erf} \left[\sqrt{\frac{5}{2}} \frac{x}{2} \right] \right\} \\ & + \sqrt{\frac{2}{5\pi}} \left[\left(\frac{31x^2}{4} + \frac{8}{5} \right) e^{-5x^2/8} + \left(\frac{x^2}{2} - \frac{8}{5} \right) e^{-5x^2/2} \right] \end{aligned} \quad (32)$$

as demonstrated in BBKS, who noted also that the asymptotic limits of this function include a cancellation to eighth order at small x , and the x^3 law expected for density maxima at large x .

The integration over x must generally be done numerically. It is worthwhile noticing that, while the exponential $\exp[-(x - \omega)^2/2(1 - \gamma^2)]$ decays rapidly to zero,

$x^n f(x)$ are monotonically and rapidly rising. As a result, the functions $G_n(\gamma, \omega)$ are sharply peaked around their maximum. For large values of ω , we find that G_0 and G_1 asymptote to

$$\begin{aligned} G_0(\gamma, \omega) & \approx \omega^3 - 3\gamma^2 \omega + B_0(\gamma) \omega^2 e^{-A(\gamma) \omega^2} \\ G_1(\gamma, \omega) & \approx \omega^4 + 3\omega^2 (1 - 2\gamma^2) + B_1(\gamma) \omega^3 e^{-A(\gamma) \omega^2}. \end{aligned} \quad (33)$$

The coefficients $A(\gamma)$, $B_0(\gamma)$ and $B_1(\gamma)$ are obtained from the asymptotic expansion of the Error function that appears in eq. (33). We have explicitly

$$A = \frac{5/2}{(9 - 5\gamma^2)}, \quad B_0 = \frac{432}{\sqrt{10\pi} (9 - 5\gamma^2)^{5/2}}, \quad B_1 = \frac{4B_0}{(9 - 5\gamma^2)}. \quad (34)$$

The rest of the calculation is easily accomplished. The two-point correlation function of peaks eventually reads

$$\xi_{\text{pk}}(\nu_1, \nu_2, r) = \left\{ \xi(r) [\bar{u}_1 \bar{u}_2 \gamma^2 + \nu_1 \nu_2 - \gamma (\nu_1 \bar{u}_1 + \nu_2 \bar{u}_2)] + \psi(r) [\bar{u}_1 \bar{u}_2 + \nu_1 \nu_2 \gamma^2 - \gamma (\nu_1 \bar{u}_1 + \nu_2 \bar{u}_2)] + \Sigma(r) [\gamma (\nu_1 \bar{u}_1 + \nu_2 \bar{u}_2) (1 + \gamma^2) - 2\gamma^2 (\bar{u}_1 \bar{u}_2 + \nu_1 \nu_2)] \right\} (1 - \gamma^2)^{-2}, \quad (35)$$

where we have introduced the mean curvature $\bar{u}(\gamma, \gamma\nu) = G_1/G_0$. The notation is such that $\bar{u}_i = \bar{u}(\gamma, \gamma\nu_i)$. The function $\bar{u}(\gamma, \gamma\nu)$ is accurately fitted by eq. (4.4) of BBKS, which is constructed to match the asymptotic large ν expansions of G_0 and G_1 given in eq. (34). In the special case $\nu_1 = \nu_2 = \nu$, the peak correlation simplifies to

$$\xi_{\text{pk}}(\nu, r) = b_\nu^2(\nu, \gamma) \xi(r) + b_\eta(\nu, \gamma) \Sigma(r) + b_\zeta^2(\nu, \gamma) \psi(r), \quad (36)$$

where the bias functions b_ν , b_η and b_ζ are

$$\begin{aligned} b_\nu(\nu, \gamma) &= \frac{\nu - \gamma \bar{u}}{1 - \gamma^2} \\ b_\eta(\nu, \gamma) &= \frac{2\gamma \nu \bar{u} (1 + \gamma^2) - 2\gamma^2 (\bar{u}^2 + \nu^2)}{(1 - \gamma^2)^2} \\ b_\zeta(\nu, \gamma) &= \frac{\bar{u} - \gamma \nu}{1 - \gamma^2}. \end{aligned} \quad (37)$$

The sign convention is chosen such that all three bias parameters are positive when $\nu \rightarrow \infty$. These functions are not independent of each other as can be seen from the relation $b_\eta = 2\gamma b_\nu b_\zeta$. Notice that b_ν is precisely the amplification factor found by BBKS when derivatives of the density correlation function are neglected.

Equation (36) is the main result of this Section. It describes the asymptotic behaviour of the peak correlation function in the limit where the correlations $\xi(r)$, $\Sigma(r)$ and $\psi(r)$ are much less than unity. Moreover, it holds for any value of the peak height ν .

IV. CLUSTERING OF DENSITY PEAKS IN GAUSSIAN INITIAL CONDITIONS

We illustrate how the peak correlation changes with the filtering, the threshold height ν and the small-scale behaviour of the transfer function. We show that the derivatives of the density correlation can boost significantly the height of the acoustic peak.

A. Biasing and the halo mass function

To gain some insight into the behaviour of the peak correlation function $\xi_{\text{pk}}(\nu, r)$, we have plotted in Fig. 2 the biasing parameters b_ν , b_η and b_ζ as a function of the

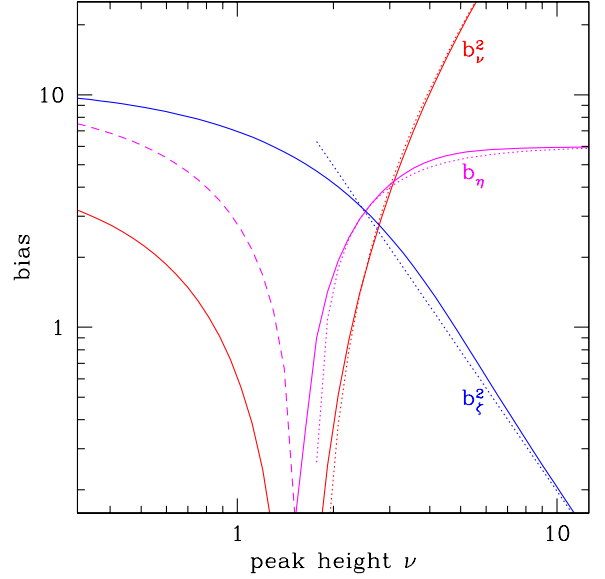


FIG. 2: Bias factors $b_\nu^2(\nu, \gamma)$, $b_\eta(\nu, \gamma)$ and $b_\zeta^2(\nu, \gamma)$ as a function of the peak height ν . The density field is smoothed on scale $R_f = 5 \ h^{-1}\text{Mpc}$ with a Gaussian filter. This leads to a correlation strength $\gamma = 0.676$. Dashed curves indicate negative values. The dotted curves are the asymptotic expansions given in eq. (38).

peak height. The density field is smoothed, as in Fig. 1, on scale $R_f = 5 \ h^{-1}\text{Mpc}$ with a Gaussian filter. The dotted curves show the following large ν approximations,

$$\begin{aligned} b_\nu(\nu, \gamma) &\approx \nu - \frac{3}{\nu} \\ b_\eta(\nu, \gamma) &\approx 6 \left(1 - \frac{3}{\nu^2} \right) \\ b_\zeta(\nu, \gamma) &\approx \frac{3}{\gamma \nu}, \end{aligned} \quad (38)$$

obtained from the asymptotic expansions of G_0 and G_1 (eq. 34). They provide a good match to the bias parameters when the peak height is larger than $\simeq 2$. As we can see, b_η tends towards the constant value of 6 when $\nu \rightarrow \infty$. Moreover, for a threshold height less than unity, b_η is negative and of absolute magnitude larger than b_ν^2 . This is also true in the intermediate region $\nu \sim 1 - 2$. For these threshold heights, both b_ν and b_η vanish while the

bias parameter b_ζ is of the order of a few. Consequently, the correlation of density maxima, albeit weak for peak heights of the order of unity, never cancels out. Overall, retaining the density correlation $\xi(r)$ solely is not a reasonable approximation when the peak height does not exceed $\nu \lesssim 4$. Although the exact value of the bias parameters changes somewhat with the smoothing scale R_f , their global behaviour varies little as γ weakly depends on the filtering scale. Therefore, the above statements hold regardless the exact amount of smoothing.

As recognised by BBKS, in the limit $\nu \gg 1$, the peak correlation is amplified by an effective bias b_ν^2 which is significantly smaller than the value ν^2/δ_{sc} derived for thresholded regions [27]. This difference arises from the correlation between the peak height ν and the peak curvature u [26, 48]. More precisely, let us consider the spherical infall model. The critical density $\delta_{\text{sc}}(z)$ for the collapse of a perturbation of size R_f and the peak height ν are related through the equality $\delta_{\text{sc}}(z) = \nu\sigma_0(R_f)$. The Lagrangian bias $b_{\text{pk}}^2 = \xi_{\text{pk}}/(\sigma_0^2\xi)$ of density peaks that are collapsing at redshift z evaluates to

$$b_{\text{pk}} \approx \frac{\nu^2 - 3}{\delta_{\text{sc}}} \quad (39)$$

when $\nu \gg 1$. This should be compared to the expression derived in [50, 53] from the Press-Schechter formalism [49, 51],

$$b_{\text{MW}} = \frac{\nu^2 - 1}{\delta_{\text{sc}}}, \quad (40)$$

In this second approach, the clustering of haloes is described by the properties of regions above a given density threshold. In both cases however, the Kaiser limit ν^2/δ_{sc} is recovered. This, however, does not apply to the bias factor derived by [56] using the ellipsoidal collapse,

$$b_{\text{ST}} \approx \frac{a\nu^2 - 1}{\delta_{\text{sc}}}, \quad (41)$$

where $a \simeq 0.7$. Assuming the peak-background split holds [27], these various bias parameters predict multiplicity functions $\nu f(\nu)$ [51] that have quite a different behaviour in the limit of large threshold heights. In particular, the Sheth-Tormen (ST) multiplicity function is proportional to $\nu \exp(-a\nu^2/2)$ [58], and exponentially deviates from the scaling inferred from b_{pk} and b_{MW} , which is $\nu f(\nu) \propto \nu^3 \exp(-\nu^2/2)$ and $\propto \nu \exp(-\nu^2/2)$, respectively. It is worth emphasising that the factor $a = 0.707$ was essentially determined by the number of massive haloes in the GIF simulations [59] and, therefore, is not a direct outcome of the ellipsoidal collapse dynamics. In fact, there is no compelling theoretical reason for a halo mass function whose high-mass end deviates exponentially from the scaling $\exp(-\nu^2/2)$. Furthermore, recent lines of evidence suggest that the high-mass tail, while being above the Press-Schechter (PS) mass function [49], may depart from the Sheth-Tormen scaling [60].

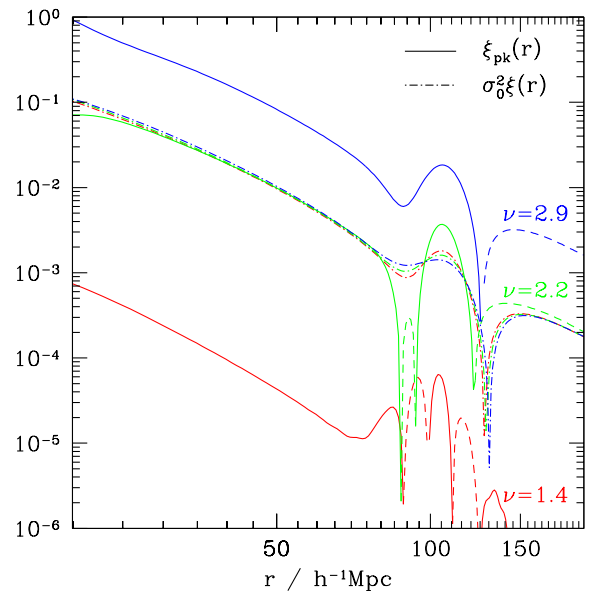


FIG. 3: The peak correlation $\xi_{\text{pk}}(\nu, r)$ (solid curves) for three different smoothing lengths $R_f = 2, 4$ and $6 h^{-1}\text{Mpc}$ (from bottom to top). These correspond to a mass scale $M_f = 9.5 \times 10^{12}, 7.6 \times 10^{13}$ and $2.6 \times 10^{14} M_\odot/h$, respectively. A peak height $\nu = \delta_{\text{sc}}/\sigma_0$ is adopted and yields the values $\nu = 1.40, 2.15$ and 2.88 , respectively. The density correlation $\sigma_0^2 \xi(r)$ is plotted as the dotted-dashed curve. Dashing indicates negative values. The acoustic signature in the peak correlation depends on the threshold height ν through the bias parameters b_ν , b_η and b_ζ . Results are shown for the ΛCDM cosmology.

In our opinion, it is likely that the true multiplicity function scales as $\exp(-\nu^2/2)$ in the limit of large ν . This would lead to a different parametrisation of the halo bias and mass function. Given the lack of a convincing physical description of these quantities, one may, for instance, consider a phenomenological bias of the form

$$b_{\text{L}} = \frac{1}{\delta_{\text{sc}}} \left(\nu^2 - c_1 + \frac{c_2}{\nu^{2p} + c_3} \right) \quad (42)$$

which, for a peak-background split, leads to a multiplicity function

$$\nu f(\nu) \propto \left(1 + \frac{c_3}{\nu^{2p}} \right)^{c_2/2pc_3} \nu^{c_1} e^{-\nu^2/2}. \quad (43)$$

For $c_1 \lesssim 3$ and $c_3 \sim c_2/c_1$ (which guarantees $b_{\text{L}} \sim 0$ in the limit $\nu \rightarrow 0$), the biasing (42) closely follows the peak scaling eq. (39) at large mass and, simultaneously, exhibits an upturn at low mass. Unfortunately, such a bias cannot be derived from an excursion set approach (upon which PS and ST are based), where $c_1 = 1$ invariably. This issue, which lies beyond the scope of the present paper, will be examined in more detail in a future work.

B. Baryon acoustic signature

We now turn to the behaviour of the peak correlation function. $\xi_{\text{pk}}(\nu, r)$ is shown in Fig. 3 for a filtering length $R_f = 2, 4$ and $6 h^{-1}\text{Mpc}$. The mass enclosed in the Gaussian window thus is $M_f = 9.5 \times 10^{12}, 7.6 \times 10^{13}$ and $2.6 \times 10^{14} M_\odot/h$, respectively. For illustration, we have adopted a peak height $\nu = \delta_{\text{sc}}(z=0)/\sigma_0$ such that $\nu = 1.4, 2.1$ and 2.9 , respectively. In the spherical infall dynamics, a tophat overdensity enclosing a similar amount of mass collapses at redshift $z \sim 0$. For comparison, the density correlation $\sigma_0^2 \xi(r)$ is also shown as the dotted curve.

The three correlations considered here exhibit a very different behaviour that reflects the strong dependence of the bias factors b_ν, b_η and b_ζ on the threshold height. In particular, we find $b_\nu = 0.057, 0.847$ and 1.771 with increasing smoothing radius. As a result, for $R_f = 2 h^{-1}\text{Mpc}$, the contribution of the term $b_\zeta^2 \psi(r)$ dominates the others and strongly suppresses the amplitude of $\xi_{\text{pk}}(\nu, r)$ relative to that of the density correlation. This term has the sign of $\psi(r)$ and features several oscillations across the BAO scale (see Fig. 1). However, for peaks of threshold height $\nu = 2.1$ identified at smoothing scale $R_f = 4 h^{-1}\text{Mpc}$, $b_\zeta^2 \psi(r)$ merely contributes to decrease the level of the minimum at distance $r \sim 90\text{--}95 h^{-1}\text{Mpc}$. Interestingly, the term $b_\eta^2 \Sigma(r)$ boosts significantly the contrast of the acoustic peak. This effect is still present, albeit weaker, for $\nu = 2.9$ and $R_f = 6 h^{-1}\text{Mpc}$. We also note that zero-crossings of $\xi_{\text{pk}}(\nu, r)$ do not generally coincide with those of $\xi(r)$, in agreement with numerical studies of the clustering of density maxima [33, 34].

Fig. 3 further illustrates the sharpening of the acoustic peak due to correlations among derivatives of the density field. The density and the peak correlations are compared in the neighbourhood of the acoustic feature for the smoothing radii $R_f = 4$ and $6 h^{-1}\text{Mpc}$ considered above. To emphasise the contrast of the acoustic peak, all the correlations have been rescaled such that, at a distance $r = 70 h^{-1}\text{Mpc}$, their amplitude is equal to 3. Fig. 3 nicely demonstrates the large impact of $b_\eta^2 \Sigma$, which fully restores the acoustic signature of $\xi_{\text{pk}}(\nu, r)$ otherwise smeared out by the large filtering. The contrast of the acoustic peak can even be enhanced relative to that of the unsmoothed ($R_f = 0.1 h^{-1}\text{Mpc}$) linear density correlation (dotted-dashed line). The effect is strongest for the density peaks identified at the smaller smoothing, $R_f = 4 h^{-1}\text{Mpc}$. For these maxima, the difference between the height of the (negative) minimum at $r \simeq 90 h^{-1}\text{Mpc}$ and the maximum at $\simeq 105 h^{-1}\text{Mpc}$ is twice as large as in the linear density correlation. The enhancement is somewhat smaller, roughly 20 per cent, for the peaks at the filtering scale $R_f = 6 h^{-1}\text{Mpc}$. This shows that density maxima behave rather differently than linearly biased tracers of the density field, whose acoustic signature cannot be larger than that of the linear matter correlation [13].

We now concentrate on the vertical lines which indi-

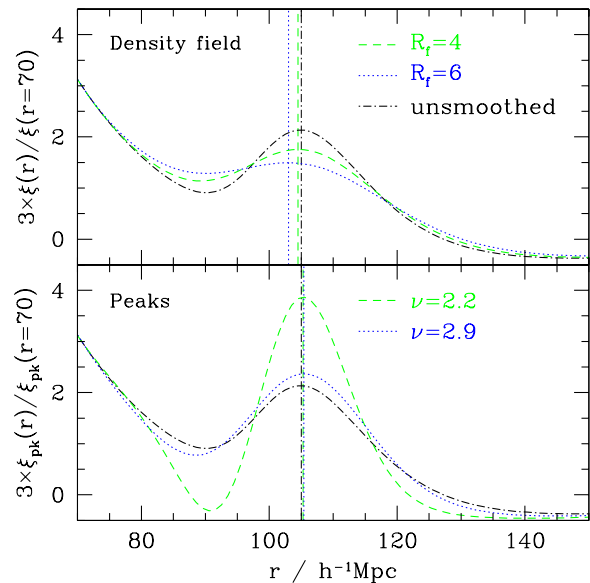


FIG. 4: A comparison between the density correlation $\xi(r)$ (top panel) and the peak correlation $\xi_{\text{pk}}(\nu, r)$ (bottom panel) around the BAO. The density field is smoothed with a Gaussian filter of width $R_f = 4$ and $6 h^{-1}\text{Mpc}$. The corresponding value of peak height is $\nu = 2.1$ and 2.9 , respectively. For clarity, all the correlations have been rescaled such that, at separation $r = 70 h^{-1}\text{Mpc}$, their amplitude is equal to 3. Also shown as the dotted-dashed line is the (unsmoothed) linear matter correlation. The vertical dashed lines indicate the position of the local maximum. The presence of $b_\eta^2 \Sigma(r)$ in the peak correlation restores, and even amplifies the acoustic peak otherwise smeared out by the large filtering. $b_\eta^2 \Sigma(r)$ also acts to reduce the shift induced by the smoothing. Results are shown for the ΛCDM cosmology.

cate the position of the local maximum. On the one hand, the top panel shows that smoothing in the density correlation generates a shift towards smaller scales, because the acoustic feature is not quite symmetric around its maximum [14]. On the other hand, the presence of $b_\eta^2 \Sigma$ in the peak correlation acts in the opposite sense and compensates for all the shift induced by the smoothing. We find the maximum to be close to its linear value $\approx 105.0 h^{-1}\text{Mpc}$. More precisely, there is a small shift of $\lesssim 0.4$ per cent towards larger scales.

C. Sensitivity to the filter shape and the transfer function

As discussed in BBKS, the filtering of the density field is an essential operation for power spectra covering a wide range of wavenumbers. However, the optimal choice of filter is disputable. Furthermore, the (analytic) properties of the filtered density field can depend significantly upon the amount of power in small-scale fluctuations. It is therefore important to assess the influence of the

smoothing operation and the small-scale transfer function on the baryon acoustic signature in the correlation of density peaks.

To this purpose, we have repeated the numerical calculation of $\xi_{\text{pk}}(\nu, r)$ using a tophat filter. To avoid divergence of the spectral moments and the correlation functions, we have introduced a high- k cutoff whose functional form is motivated by the damping of fluctuations due to the free streaming of the dark matter particle(s). So far, we have considered a CDM cosmology in which the velocity dispersion of the dark matter particle is negligible. By contrast, in Warm Dark Matter (WDM) cosmologies, the dark matter candidate(s) can suppress the matter power spectrum on galaxy scales $r \sim 0.1 h^{-1}\text{Mpc}$ [61]. The latter can be approximated as $P_{\text{WDM}}(k) = T^2(k)P_{\text{CDM}}(k)$, where the transfer function that accounts the free-streaming cutoff has the form [62]

$$T(k) = \left[1 + (\alpha k)^{2p}\right]^{-5/p}. \quad (44)$$

Here, $p \approx 1.12$ and α depends upon the properties of the dark matter particles. Typically, $0.01 \lesssim \alpha \lesssim 0.1$ for thermal relics of mass $\sim 1 - 10 \text{ keV}$.

In spite of its compactness, the tophat filter has some inconvenient. Firstly, because of its slowly decaying tail, it produces a density field that is not differentiable for generic CDM power spectra [51]. Secondly, it is good at discriminating peaks from the background field so long as the height of the latter is small, namely, when the background field is uncorrelated over scales comparable to the filtering length. By contrast, the Gaussian filter is less sensitive to high frequencies and thus fares better at picking up smoother objects. Indeed, the “true” filter may lie between these two extremes [48]. Notice that the sharp k -space window will not be considered here as it leads to undesirable oscillations at all separations.

Fig. 5 shows the baryon acoustic peak in the correlation of density maxima for the smoothing radii used in Fig. 3 and 4. Note, however, that the filter mass scale is now roughly four times smaller than with the Gaussian window. The peak correlation is plotted for two values of the free-streaming cutoff, $\alpha = 0.01$ and 0.1 (top and bottom panels). Also shown in both panels for comparison is the linear matter correlation (dotted-dashed line). At smoothing length $R_f = 4$ and $6 h^{-1}\text{Mpc}$, the enhancement of the acoustic peak is very significant for $\alpha = 0.1$ while, for $\alpha = 0.01$, it is only 10-20 per cent. The main reason is a sharper power spectrum, which leads to a larger contribution of the correlations $\Sigma(r)$ and $\psi(r)$ to $\xi_{\text{pk}}(\nu, r)$. At $R_f = 4 h^{-1}\text{Mpc}$ for instance, the spectral width is $\gamma = 0.48$ and 0.26 for $\alpha = 0.1$ and 0.01 , respectively. This difference mostly arises because of the second spectral moment, which increases from $\sigma_2 = 0.40$ to 0.76 upon the decrease in the free-streaming scale. Yet another interesting feature of Fig. 5 is the rather broad acoustic peak at filtering scale $R_f = 2 h^{-1}\text{Mpc}$ (see bottom panel), for which $\nu = 0.96$. This broadening follows from the fact that b_η is negative at that value of thresh-

old height. As a consequence, the oscillatory pattern of $\Sigma(r)$ across the BAO (cf. Fig. 1 for instance) smears out the acoustic feature in $\xi_{\text{pk}}(\nu, r)$. As seen from Fig. 2, this damping always occurs at sufficiently low values of the threshold height, $\nu \lesssim 1$ regardless of the filtering length. It should also be noted that, unlike the correlation of density maxima, the BAO in the smoothed linear matter correlation is weakly insensitive to the small-scale behaviour of the power spectrum. In $\xi_{\text{pk}}(\nu, r)$ however, the BAO acquires an extra dependence upon the high- k tail of the transfer function through the correlations $\Sigma(r)$ and $\psi(r)$.

To summarise,

- Both $\Sigma(r)$ and $\psi(r)$ contribute to the correlation of density maxima and can affect the shape of the baryon acoustic signature for peak heights $\nu \lesssim 4$.
- $\psi(r)$ makes a significant contribution only in the range $1 \lesssim \nu \lesssim 2$, where b_ν and b_η are much less than unity.
- The contribution of $\Sigma(r)$ increases with the spectral width γ . At constant filtering length, it increases with the amount of power suppression due to the small scale free streaming.
- b_η is positive (negative) for $\nu \gtrsim 1$ ($\nu \lesssim 1$). As a result, the baryon acoustic peak is generally enhanced in $\xi_{\text{pk}}(\nu, r)$ when $\nu \gtrsim 1$, and damped out when $\nu \lesssim 1$.

These results depend upon the exact shape of the filter and the transfer function. Clearly however, the effect cannot be reduced to a simple rescaling of the linear matter correlation.

Thus far, we have explored the BAO signature in the correlation of maxima of the primordial density field. However, pairwise motions caused by small- and large-scale structures, redshift space distortions etc. are likely to degrade the acoustic signature, leading to a broadening and, possibly, a shift of the acoustic peak. A thorough investigation of these effects is postponed to a subsequent paper. In the next Section, we will nonetheless demonstrate the cancellation of the mean streaming of peak pairs at leading order. This indicates that the impact of nonlinearities on the acoustic signature of density maxima should be small. Namely, some of the initial enhancement of the BAO may survive in the clustering of high redshift density peaks. Furthermore, we expect shifts no larger than those found in recent studies of the baryon acoustic oscillations [11, 12, 13, 14, 16, 17, 20].

V. MEAN STREAMING OF PEAK PAIRS

The time evolution of the peak correlation function ξ_{pk} is governed by the following pair conservation equation (so long as peaks do not merge),

$$\frac{\partial \xi_{\text{pk}}}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 (1 + \xi_{\text{pk}}) v_{12}(r) \right], \quad (45)$$

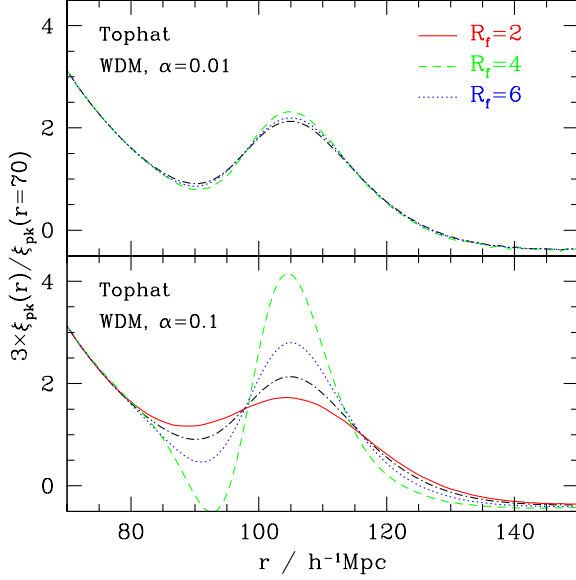


FIG. 5: The correlation of density maxima that trace the density field smoothed with a Tophat filter. The smoothing radii $R_f = 2, 4$ and $6 \ h^{-1}\text{Mpc}$ correspond to a mass $M_f = 2.5 \times 10^{12}, 2 \times 10^{13}$ and $6.8 \times 10^{13} \ M_\odot/h$, respectively. Results are shown for a WDM power spectrum with a cutoff scale $\alpha = 0.01$ and $0.1 \ h^{-1}\text{Mpc}$ (see text). The peak height is chosen such that $\nu = \delta_{\text{sc}}/\sigma_0$, as before. In both panels, the dotted-dashed curve is the linear matter correlation. The peak correlation $\xi_{\text{pk}}(\nu, r)$ for $\alpha = 0.01$ and $R_f = 2$ is not shown as it is too much affected by numerical noise.

where $v_{12}(r)$ is the average, number weighted pairwise velocity $[\mathbf{v}_{\text{pk}}(\mathbf{x}_2) - \mathbf{v}_{\text{pk}}(\mathbf{x}_1)] \cdot \hat{\mathbf{r}}$ along the line of sight, or mean streaming [67, 68]. The rest of this Section demonstrates that the relative pairwise velocity of peaks vanishes at first order.

A. Zeldovich approximation

The Eulerian comoving position and velocity of a density peak can generally be expressed as a mapping

$$\mathbf{x}_{\text{pk}} = \mathbf{q} + \mathbf{S}(\mathbf{q}, a), \quad \mathbf{v}_{\text{pk}} = \dot{\mathbf{S}}(\mathbf{q}, a), \quad (46)$$

where \mathbf{q} is the initial position, $\mathbf{S}(\mathbf{q}, a)$ is the displacement field and a is the scale factor. A dot denotes a time derivative. At first order, the peak position is described by the Zeldovich approximation [63], in which the displacement factorises into a time and a spatial component,

$$\mathbf{S} = -D(a)\nabla\Phi(\mathbf{q}), \quad \dot{\mathbf{S}} = -\beta(a)\nabla\Phi(\mathbf{q}). \quad (47)$$

Here, $\Phi(\mathbf{q}) = \phi(\mathbf{q}, a)/4\pi G\bar{\rho}_m(a)a^2D_+(a)$ is the perturbation potential, $\phi(\mathbf{q}, a)$ is the Newtonian gravitational potential, $\bar{\rho}_m(a)$ is the average matter density and $D_+(a)$ is the linear growth factor. $\beta(a) = HD_+f$ is proportional to the logarithmic derivative of the growth factor,

$f = d\ln D_+/d\ln a$, which scales as $f(a) \approx \Omega_m(a)^{0.6}$ for a wide range of CDM cosmologies [64].

Such a simple model cannot account for the internal properties of peaks for example [65]. Nevertheless, it is not intended to be realistic, but only to capture the weakly nonlinear regime so as to exemplify the basic effect. A more sophisticated approach can be found in [66] for instance. The two-point correlation of Eulerian peak positions $\mathbf{x}_{\text{pk}}(\mathbf{q})$ is now obtained from the statistics of the velocity field $-\nabla\Phi(\mathbf{q})$. The complication arises from the fact that the latter has to be evaluated at those maxima of the density field.

B. Correlations of velocity field

Let us introduce the normalised comoving velocity $\mathbf{u}(\mathbf{q}) = \mathbf{v}(\mathbf{q})/(\beta\sigma_{-1})$. The rms variance σ_{-1} is related to the present-day, three-dimensional velocity dispersion of random field points, which is $\sim 350 \text{ km s}^{-1}$ in the cosmology considered here. Also, the notational shorthand $\Delta\mathbf{u}$ will designate the difference $\mathbf{u}(\mathbf{q}_2) - \mathbf{u}(\mathbf{q}_1)$. The auto-correlation of the velocity and its cross-correlations with the fields η_i, ν and ζ_{ij} can be written as

$$\begin{aligned} \langle v_i(\mathbf{q}_1)v_j(\mathbf{q}_2) \rangle &= \mathcal{U}_1(r)\hat{r}_i\hat{r}_j + \mathcal{U}_2(r)\delta_{ij} \\ \langle v_i(\mathbf{q}_1)\eta_j(\mathbf{q}_2) \rangle &= \gamma_\nu \xi_1(r)\hat{r}_i\hat{r}_j + \gamma_\nu \xi_2(r)\delta_{ij} \\ \langle v_i(\mathbf{q}_1)\nu(\mathbf{q}_2) \rangle &= \mathcal{V}(r)\hat{r}_i \\ \langle v_i(\mathbf{q}_1)\zeta_{lm}(\mathbf{q}_2) \rangle &= \mathcal{S}_1(r)\hat{r}_i\hat{r}_l\hat{r}_m + \mathcal{S}_2(r)(\hat{r}_i\delta_{lm} + \hat{r}_l\delta_{im} + \hat{r}_m\delta_{il}). \end{aligned} \quad (48)$$

Here $v_i(\mathbf{q})$ designates the components of $\mathbf{u}(\mathbf{q})$. For sake of completeness, the various angle average correlations are

$$\begin{aligned} \mathcal{U}_1(r) &= -\frac{1}{\sigma_{-1}^2} \int_0^\infty d\ln k k^{-2} \Delta^2(k) j_2(kr) \\ \mathcal{U}_2(r) &= \frac{1}{\sigma_{-1}^2} \int_0^\infty d\ln k k^{-2} \Delta^2(k) \left[\frac{1}{3} j_0(kr) + \frac{1}{3} j_2(kr) \right] \\ \xi_1(r) &= -\frac{1}{\sigma_0^2} \int_0^\infty d\ln k \Delta^2(k) j_2(kr) \\ \xi_2(r) &= \frac{1}{\sigma_0^2} \int_0^\infty d\ln k \Delta^2(k) \left[\frac{1}{3} j_0(kr) + \frac{1}{3} j_2(kr) \right] \\ \mathcal{V}(r) &= -\frac{1}{\sigma_{-1}\sigma_0} \int_0^\infty d\ln k k^{-1} \Delta^2(k) j_1(kr) \\ \mathcal{S}_1(r) &= -\frac{1}{\sigma_{-1}\sigma_2} \int_0^\infty d\ln k k \Delta^2(k) j_3(kr) \\ \mathcal{S}_2(r) &= \frac{1}{\sigma_{-1}\sigma_2} \int_0^\infty d\ln k k \Delta^2(k) \left[\frac{1}{5} j_1(kr) + \frac{1}{5} j_3(kr) \right] \end{aligned} \quad (49)$$

for the Gaussian density field considered here. The functions ξ_1 and ξ_2 satisfy the relation $\xi_1(r) + 3\xi_2(r) = \xi(r)$. Like the spectral width γ , the parameter $\gamma_\nu = \sigma_0^2/(\sigma_{-1}\sigma_1)$ characterises the range over which the velocity power spectrum $\propto k^{-2}\Delta^2(k)$ is large. It is worthwhile noticing that the latter peaks on scale much larger than

the density power spectrum. Also, the correlation $\mathcal{V}(r)$ is proportional to the mean streaming of ambient field points,

$$\langle [1 + \delta(\mathbf{q}_1)] [1 + \delta(\mathbf{q}_2)] \Delta \mathbf{u} \cdot \hat{\mathbf{r}} \rangle = 2\sigma_0 \mathcal{V}(r), \quad (50)$$

which is mass weighted by the densities at \mathbf{q}_1 and \mathbf{q}_2 .

C. Mean streaming at leading order

The calculation of the peak pairwise velocity is more intricate than the peak correlation since we have three additional degrees of freedom, but it closely follows the analysis described in Section §III.

The line of sight pairwise velocity weighted over all pairs with separation r can be expressed as

$$[1 + \xi_{\text{pk}}(r)] v_{12}(r) = \langle n_{\text{pk}} \rangle^{-2} \times \frac{1}{4\pi} \int d\Omega_{\hat{\mathbf{r}}} d\mathbf{y}_1 d\mathbf{y}_2 (\Delta \mathbf{u} \cdot \hat{\mathbf{r}}) n_{\text{pk}}(\mathbf{q}_1) n_{\text{pk}}(\mathbf{q}_2) P(\mathbf{y}_1, \mathbf{y}_2; r) \quad (51)$$

The local peak density $n_{\text{pk}}(\mathbf{q})$ is given by equation (7), supplemented by the appropriate conditions to select those maxima with a certain threshold height. To obtain the average pair velocity as a function of separation r , we need to calculate the two-point probability distribution for the variables $\mathbf{y}^\top = (v_i, \eta_i, \nu, \zeta_A)$. At zero lag, both v_i and η_i are uncorrelated with the density and the Hessian ζ_A . Hence, the covariance M_1 of the components (v_i, η_i, ν) is a 7×7 block matrix which reads

$$M_1 = \begin{pmatrix} 1/3 \mathbf{I} & \gamma_v/3 \mathbf{I} & 0_{3 \times 1} \\ \gamma_v/3 \mathbf{I} & 1/3 \mathbf{I} & 0_{3 \times 1} \\ 0_{1 \times 3} & 0_{1 \times 3} & 1 \end{pmatrix}. \quad (52)$$

Similarly, the covariance M_2 of the Hessian, and the cross-covariance M_3 between ζ_A and the entries (v_i, η_i, ν) are

$$M_2 = \begin{pmatrix} A/15 & 0_{3 \times 3} \\ 0_{3 \times 3} & I/15 \end{pmatrix} \quad (53)$$

$$M_3 = \begin{pmatrix} 0_{3 \times 3} & 0_{3 \times 3} & -\gamma/3 \mathbf{1}_{3 \times 1} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 1} \end{pmatrix}.$$

Proceeding as in §III, we now consider the regime where all the correlations are much less than unity. Expanding the two-point probability distribution $P(\mathbf{y}_1, \mathbf{y}_2; \mathbf{r})$ in the small perturbation $B(\mathbf{r})$, we obtain

$$e^{-Q(\mathbf{y}_1, \mathbf{y}_2; \mathbf{r})} \approx (1 + \mathbf{y}_1^\top M^{-1} B M^{-1} \mathbf{y}_2) e^{-\bar{Q}(\mathbf{y}_1, \mathbf{y}_2)}. \quad (54)$$

As before, the 13×13 matrix $B(\mathbf{r})$ denotes the covariances at different comoving positions. It has a (unique) harmonic decomposition in term of the matrices $B_i^{\ell, m}$ (equation (12)). The computation of these matrices is, however, unnecessary as we will see later. Furthermore, the quadratic form $\bar{Q}(\mathbf{y}_1, \mathbf{y}_2)$ now reads

$$2\bar{Q} = \frac{3v_1^2}{1 - \gamma_v^2} + \nu_1^2 + \frac{(\gamma\nu_1 + \text{tr}\zeta_1)^2}{1 - \gamma^2} + \frac{5}{2} \left[3\text{tr}(\zeta_1^2) - (\text{tr}\zeta_1)^2 \right] + 1 \leftrightarrow 2. \quad (55)$$

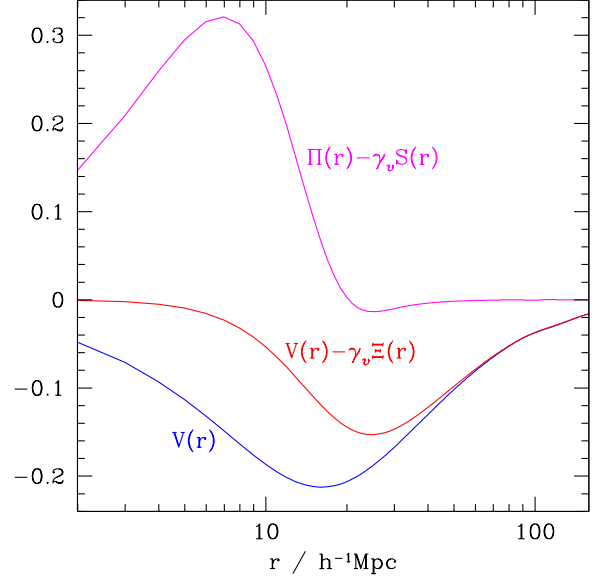


FIG. 6: The correlations that contribute to the leading order mean streaming of peak pairs, equation (64). These are compared to the line of sight pairwise velocity $\mathcal{V}(r)$ of ambient field points. Results are shown at a filtering scale $R_f = 5 h^{-1} \text{Mpc}$. The correlation $\mathcal{V} - \gamma_v \Xi$ is strongly damped on scales less than the characteristic inter-peak distance $\propto R_f$, but the term $\Pi - \gamma_v S$ can significantly restore the small-scale streaming motions when the peak height is $\nu \lesssim 3$. On large scales, the mean streaming of density maxima is unaffected by small-scale exclusion effects and closely follows that of random field points.

We note that the velocity dispersion of density maxima is lower by a factor $1 - \gamma_v^2$ than that of random field points [26]. One has $\gamma_v \approx 0.43$ for a smoothing length $R_f = 5 h^{-1} \text{Mpc}$. Moreover, eq. (55) leads to a one-point probability distribution $P(\mathbf{y}|\text{peak}) \propto \exp[-\bar{Q}(\mathbf{y})]$ separable into the product $P_v(v_i)P_{\nu\zeta}(\nu, \zeta_A)$, where $P_{\nu\zeta}$ is the one-point distribution of the density and its second derivatives, and $P_v(v_i)$ is the velocity distribution of peaks,

$$P_v(v_i|\text{peak}) = \frac{3^{3/2}}{(2\pi)^{3/2} (1 - \gamma_v^2)^{3/2}} \exp \left[-\frac{3v^2}{2(1 - \gamma_v^2)} \right]. \quad (56)$$

The separability of the one-point distribution separability considerably simplifies the calculation.

Taking the product $(\Delta \mathbf{u} \cdot \hat{\mathbf{r}}) B(\mathbf{r})$ mixes the various multipole matrices $B_i^{\ell, m}$, so that the result depends on the correlation functions of v_i, η_i, ν and ζ_A in a rather complicated way. Averaging over the directions gives

$$\tilde{B} = \frac{1}{4\pi} \int d\Omega_{\hat{\mathbf{r}}} (\Delta \mathbf{u} \cdot \hat{\mathbf{r}}) M^{-1} B M^{-1} = \begin{pmatrix} \tilde{B}_1 & \tilde{B}_3^\top \\ \tilde{B}_3 & \tilde{B}_2 \end{pmatrix}, \quad (57)$$

where the block matrices \tilde{B}_i have the same dimensions as M_i . Owing to the angular average, the calculation of

the $B_i^{\ell,m}$ can be avoided by writing down the entries of $B_i(\mathbf{r})$ using the relations eq. (3) and (50), and retaining only those components involving odd products of the unit vector $\hat{\mathbf{r}}_i$. A tedious calculation shows that $\tilde{B}_1(r)$ and $\tilde{B}_3(r)$ can be cast into the form

$$\begin{aligned}\tilde{B}_1 &= \begin{pmatrix} 0_{3 \times 3} & 0_{3 \times 3} & \alpha_1 \Delta \mathbf{u} \\ 0_{3 \times 3} & 0_{3 \times 3} & \alpha_2 \Delta \mathbf{u} \\ \alpha_1 \Delta \mathbf{u}^\top & \alpha_2 \Delta \mathbf{u}^\top & 0 \end{pmatrix} \\ \tilde{B}_3 &= \begin{pmatrix} \gamma \alpha_1 \Upsilon_1 + 3\alpha_3 \Upsilon_2 & \gamma \alpha_2 \Upsilon_1 + 3\alpha_4 \Upsilon_2 & 0_{3 \times 1} \\ & 3\alpha_3 \Upsilon_3 & \\ & & 3\alpha_4 \Upsilon_3 & 0_{3 \times 1} \end{pmatrix},\end{aligned}\quad (58)$$

The functions $\alpha_i(r)$ are

$$\begin{aligned}\alpha_1(r) &= \frac{\mathcal{V} - \gamma_v \Xi + \gamma (\Pi - \gamma_v \mathcal{S})}{(1 - \gamma^2)(1 - \gamma_v^2)} \\ \alpha_2(r) &= \frac{\Xi - \gamma_v \mathcal{V} + \gamma (\mathcal{S} - \gamma_v \Pi)}{(1 - \gamma^2)(1 - \gamma_v^2)} \\ \alpha_3(r) &= \frac{\Pi - \gamma_v \mathcal{S}}{1 - \gamma_v^2} \\ \alpha_4(r) &= \frac{\mathcal{S} - \gamma_v \Pi}{1 - \gamma_v^2},\end{aligned}\quad (59)$$

where we have omitted the explicit r -dependence of the correlations for brevity. The 3×3 matrices Υ_i have the components Δv_i of the vector $\Delta \mathbf{u}$ as entries,

$$\begin{aligned}\Upsilon_1 &= \begin{pmatrix} \Delta v_1 & \Delta v_2 & \Delta v_3 \\ \Delta v_1 & \Delta v_2 & \Delta v_3 \\ \Delta v_1 & \Delta v_2 & \Delta v_3 \end{pmatrix} \\ \Upsilon_2 &= \begin{pmatrix} \Delta v_1 & 0 & 0 \\ 0 & \Delta v_2 & 0 \\ 0 & 0 & \Delta v_3 \end{pmatrix} \\ \Upsilon_3 &= \begin{pmatrix} \Delta v_2 & \Delta v_1 & 0 \\ \Delta v_3 & 0 & \Delta v_1 \\ 0 & \Delta v_3 & \Delta v_2 \end{pmatrix}.\end{aligned}\quad (60)$$

We have also set

$$\Pi(r) = \Pi_1 + 5\Pi_2, \quad \mathcal{S}(r) = \mathcal{S}_1 + 5\mathcal{S}_2. \quad (61)$$

The matrix \tilde{B}_2 is identically zero.

The rest of the calculation is easily accomplished owing to the factorisation of the one-point probability distribution $P(\mathbf{y}|\text{peak})$. Notice that the scalar $\mathbf{y}_1^\top \tilde{B} \mathbf{y}_2$ contains terms linear and quadratic in \mathbf{u}_1 and \mathbf{u}_2 . After integrating out the velocities, the linear terms vanish and we find

$$\begin{aligned}\int d^3 \mathbf{u}_1 d^3 \mathbf{u}_2 \mathbf{y}_1^\top \tilde{B} \mathbf{y}_2 P(\mathbf{u}_1|\text{peak}) P(\mathbf{u}_2|\text{peak}) &= \\ [\alpha_1 (\nu_1 - \nu_2) + (\gamma \alpha_1 + \alpha_3) (\text{tr} \zeta_1 - \text{tr} \zeta_2)] (1 - \gamma_v^2) &.\end{aligned}\quad (62)$$

Transforming to the set of variables (u_i, v_i, w_i) and using the expressions of the bias parameters b_ν and b_ζ , eq. (37), the mean streaming of peak pairs can be recast as

$$\begin{aligned}(1 + \xi_{\text{pk}}) v_{12} &= [b_\nu(\nu_1, \gamma) - b_\nu(\nu_2, \gamma)] (\mathcal{V} - \gamma_v \Xi) \\ &+ [b_\zeta(\nu_1, \gamma) - b_\zeta(\nu_2, \gamma)] (\Pi - \gamma_v \mathcal{S}).\end{aligned}\quad (63)$$

It should be kept in mind that the pairwise velocity is weighted by the number density of peaks at \mathbf{q}_1 and \mathbf{q}_2 .

D. A first order cancellation

Equation (64) demonstrates the cancellation of the mean pairwise velocity of peak pairs at leading order when $\nu_1 = \nu_2$. For $\nu_1 > \nu_2 \gtrsim 1$, there is a net flow whose sign depends upon the detailed behaviour of the correlations $\mathcal{V} - \gamma_v \Xi$ and $\Pi - \gamma_v \mathcal{S}$. As seen in Fig. 6, the former is negative on scales $r \lesssim 200 h^{-1} \text{Mpc}$, while the latter contributes mostly at small scales $r \lesssim 20 h^{-1} \text{Mpc}$ where it is positive. Therefore, this leads to a negative net flow owing to the monotonic increase (decrease) of $b_\nu(\nu, \gamma)$ ($b_\zeta(\nu, \gamma)$) with the peak height. This “inward transport” reflects the fact that small peaks tend to accrete onto high density regions.

The mean streaming $\mathcal{V}(r)$ of random field points is also shown in Fig. 6 for comparison. Peak-peak exclusion leads to a deficit of pairs at separation $\lesssim R_\star$ comparable to the filtering scale. Consequently, one would naively expect modes with wavelength larger than R_f alone to contribute to the relative velocity of peaks. This is the reason why the correlation $\mathcal{V} - \gamma_v \Xi$ is strongly suppressed relative to $\mathcal{V}(r)$ on scales less than the filtering length. Interestingly however, the term $\Pi - \gamma_v \mathcal{S}$, which is largest at distances of the order of the smoothing length, can restore significantly the small-scale mean streaming when the threshold height is less than $\nu \lesssim 3$ (for which $b_\zeta \gtrsim b_\nu$). We also note that exclusion effects modify the pairwise velocity out to distances that are much larger than the typical extent $\sim R_f$ of density maxima. At large enough separations $r \gtrsim 50 h^{-1} \text{Mpc}$ however, the mean streaming of peak pairs is unaffected by small-scale exclusion effects and closely tracks the pairwise velocity $\mathcal{V}(r)$ of ambient field points.

The first order cancellation of the mean streaming of peak pairs appears consistent with the analysis of [13], who derived a similar result for the pairwise displacement of linearly biased tracers of the density field. We thus expect changes in the acoustic signature of $\xi_{\text{pk}}(r)$ due to nonlinear motions to be roughly at the few percent level. Finally, it is worthwhile noticing that, in addition to the damping from the nonlinear propagator [69], Renormalised Perturbation Theory predicts a mode-coupling correction which is also second order in the linear matter correlation [16]. It would be interesting to ascertain whether the nonlinear correction to the peak correlation can be recast in a similar way.

VI. CONCLUSION

We have investigated the strength of the baryon acoustic signature in the two-point correlation of maxima of the linear (Gaussian) density field. To this purpose, we examined in Section §III the large-scale asymptotics of the peak correlation $\xi_{\text{pk}}(r)$ and derived the leading order contribution, eq. (36). In contrast to the analysis of BBKS, spatial derivatives of the linear density correlation $\xi(r)$ were included in our derivation. These derivatives

are not negligible for generic CDM power spectra, especially around the BAO scale where they exhibit large oscillations. The leading asymptotic behaviour of the peak correlation is governed by three terms : a term previously derived in BBKS plus two terms involving the spatial derivatives $\Sigma(r)$ and $\psi(r)$ of the linear density correlation. The relative contribution of these functions is governed by three bias parameters b_ν , b_η and b_ζ (eq. 37).

In Section §IV, we demonstrated that these extra terms can have a large impact on the correlation of density maxima in the vicinity of the BAO. The results are sensitive to the exact value of the threshold height ν , the smoothing length R_f , the filter shape and the high- k tail of the transfer function. For the Gaussian filter adopted throughout this paper, the contrast of the baryon acoustic signature can be significantly enhanced relative to that in the linear matter correlation when the peak height is in the range $1 \lesssim \nu \lesssim 3$. This boost originates from the oscillatory behaviour of $\Sigma(r)$ and $\psi(r)$ around the sound horizon scale. For instance, we find that, at filtering scale $M_f = 8 \times 10^{13} M_\odot/h$, the contrast of the BAO in the correlation of density maxima of height is about twice as large as in $\xi(r)$. The amplification fades as we go to larger peak height. For a peak height of the order of unity, $\xi_{\text{pk}}(r)$ can exhibit several bumps which reflect those of $\psi(r)$ around the BAO scale. For a threshold height less than $\lesssim 1$, the original acoustic peak is smeared out by the negative contribution of the term $b_\eta \Sigma(r)$.

To avoid the divergence of the (fourth order) spatial derivative $\psi(r)$ of the density correlation, we have filtered the density field with a Gaussian window. The main drawback of this filtering is the lack of a well-defined mass and spatial extent associated to the density fluctuations. A tophat filter appears better motivated in the context of, e.g., the spherical infall model, although it does not produce an infinitely differentiable density field for generic CDM power spectra. Furthermore, the differentiability of the density field depends strongly upon the small-scale behaviour of the transfer function. Fluctuations in the matter density are damped on scales smaller than the free-streaming length of the dark matter particle. In the CDM cosmology considered here, the velocity dispersion of the dark matter particle is negligible. By contrast, Warm Dark Matter (WDM) particles such as massive neutrinos for example can suppress the matter power spectrum on galaxy scales [61]. For these reasons, we also discussed in Section §IV how the BAO changes with the window function and the free-streaming cutoff. We found that the correlation of density maxima is more sensitive to the properties of the dark matter particle(s) than the matter correlation itself. This follows from the dependence of the clustering properties of peaks upon the derivatives of the density field. However, whether the baryon acoustic signature in the clustering of peaks varies significantly with the nature of dark matter remains to be determined.

We emphasise that the calculations presented in this paper are performed in the initial conditions. As non-

linearities progress, the late-time acoustic signature is smeared out by structure formation as reported by many authors using N-body simulations [7]. This might explain why numerical investigations of the clustering of dark matter haloes do not show any evidence for an amplification of the BAO. Clearly, it will be important to translate the Lagrangian analysis into quantitative predictions for the baryon oscillation in the dark matter haloes distribution. In this respect, the ability of the Zeldovich approximation used in Section §V provides a rather limited description of the late-time distribution of density maxima such as cluster- or galaxy-size haloes [23, 70]. One should clearly rely upon higher order Lagrangian schemes.

Notwithstanding this, we demonstrated in §V the cancellation of the mean streaming of peak pairs at leading order. This result is consistent with the analysis of [13] who considered the pairwise displacement of linearly biased tracers of the density field, and with the findings of [16] who explored the impact of nonlinearities on the linear matter correlation. Hence, the acoustic signature in the clustering of density maxima appears also robust to nonlinearities. Some of the initial enhancement of the BAO may thus survive in the clustering of high redshift density peaks. Furthermore, we expect the shifts in the late-time correlation of peak to be no larger than those found in the matter correlation [12, 14, 16, 17, 20].

Since the sound horizon scale will primarily be measured through the clustering of galaxies, it would be valuable to explore models more sophisticated than the local bias model used so far [9, 13, 14], so as to ascertain how much of the amplification of the acoustic signature in the initial clustering of density maxima propagates into the late-time correlation of galaxies and clusters. The effect of galaxy biasing will be examined in a separate paper. In this respect, it is worthwhile noticing that the clustering of the SDSS LRG sample [2], for instance, shows a sharper acoustic peak than expected from linear theory and smearing due to nonlinearities. However, one should remember that the data points are strongly correlated, so that a very high acoustic peak is actually allowed by the current Λ CDM cosmology with 1σ error when the full covariance matrix is considered. While there is certainly a direct correspondence between the most massive clusters in the evolved matter distribution and the largest maxima of the initial density field, it is still unclear how well galaxies trace density peaks given that dark matter haloes themselves do not form exactly out of the initial density peaks [71]. As a general criterion, the identification scheme should reproduce the observed properties of the galaxy distribution. Future redshift surveys such as ADEPT, BOSS, CIP, DES, HETDEX, LSST, WiggleZ or WFMOS [72], which will obtain redshifts for millions of galaxies, should achieve an exquisite precision on the shape of the baryon acoustic signature in the clustering of galaxies. Beyond the nature of dark energy, a precise measurement of the BAO could also place constraints on galaxy biasing and the physical mechanisms that cause

it.

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